Contextual versus Absolute Coordination - Step One*

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Abstract

An agent who is interested in reading [Ago01a] may also be asked for looking at alternative paradigms of cooperation but "contextual coordination" of [Ago01a, Sec. 3]. We do it in this paper by introducing some paradigms of "absolute," model-theoretic coordination. We argue by using examples and proofs how these paradigms provide an uniform first-order framework to eventually investigate coordination problems and scenarios of interest in a variety of research areas and domains.

Keywords: Formalisms and logics for agents and MAS; coordination.

1 Motivations and Results

Coordination systems and theories can be divided into three broad classes: those based on convention, those based on communication, and those based on learning. Some example in the first class was given in [ST92b, ST92a], where social laws [Lew69] are imposed by the system designer (see also [PAM97, ST97]) so that optimal joint action is assured. In the second class, agents' coordination is based on communication (see for instance [Wei93, WJ95]). This second class might be thought as a special case of the normative class, where the communication language is assumed to be the convention. So, what makes this class different is rather the emphasis it gives to the communicative agents' skills with regard to agents' coordination problems. In this class, it does make sense to speak about *failure messages* that prevent the agents from coordinating. (see for instance [WJ95] for some further remarks and references on the influence of *speech act theory* in communication). In the third class, coordination might be learned through repeated interaction; see for instance [Bou96, BGS$^+$91, SSH94, Wei93].

Cooperation has been extensively studied, both in game theory and AI, where fully cooperative problems arise in task distribution as well as within the historically older sub-area of the multi-agent problem solving. An incomplete list is [Dur88, Kra97, PAM97, WJ94, WJ99].

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The key feature of coordination and the like dynamics is that these embrance all the most fundamental characteristics of (strategic) interactions in a large spectrum of applications and areas, involving both individual and coalitional decision making. To model coordination processes, it is required modeling the agents' abilities, knowledge and beliefs as well as the environment, which is possibly made unpredictable by the agents behavior itself. This makes coordination and similar strategic dynamics ideal paradigms for modeling, developing and testing general techniques and architectures needed in distributed multi-agent systems, as well as in the emergent area of distributed knowledge management (see for instance [BBM00] and the references cited there).

Regarding theories and techniques, we are convinced that inductive methods are a very promising approach to handle the learning requirements such complex systems have to deal with. In contrast to deductive approaches, that usually impose very strong informational assumptions (*e.g.* rationality, knowledge) at the individual decision-making level, so that are widely recognized as having serious deficiencies (see for instance [Bin87, Bin88]), inductive methods seem more appropriate, at least in the view of [Car66]: "The observations we make in everyday life as well as the more systematic observations of science reveal certain repetitions or regularities in the world...The laws of science are nothing more than statements expressing these regularities as precisely as possible." (p. 3)

Learning is thus a central theme of research in understanding coordination processes, since to coordinate a decision maker has often to discover the strategy, or sometimes the beliefs adopted by her partner in the process. In this context, in all our work we focus on developing inductive representations and algorithms that allow possibly 'rational', belief-based decision makers following those representations and algorithms to reliably and efficiently coordinate in several, different strategic situations.

To investigate the fundamental challenges of such complex dynamics, theories and systems, in this paper we are interested in certain coordination games. We call these games **paradigms**. In particular, we focus on coordination paradigms where the agents always move simultaneously and all relevant moves are made by the agents—agents' moving components are functions in mathematical sense; no randomness ever intervenes. These paradigms are all based on pairwise communication, so that the kind of models we discuss is suitable for modeling group situations where communication is not with the whole group, as in an auction, but pairwise, as happens for instance in commercial transactions.

Our framework provides an uniform treatment of coordination, both absolute and contextual, and allows us to answer some sample problems. We prove the equivalence of two kind of contextual coordination, where agents are or are not allowed to input the guesses by an agent solving a problem. Our proof is constructive, in the sense that it provides an uniform map Γ from agents to agents such that for all the agents in one paradigm and for all coordination contexts, the agents coordinate pairwise in one paradigm iff the transformed-by-Γ agents pairwise coordinate in the second paradigm. We also introduce two kinds of "forgiving" and "blind" agents, and show that every forgiving agent able to always play consistently with his or her beliefs is blind, but that the converse is false, that is, there there are blind agents who are not forgiving. Our last result (Corollary (26)) says that the coordination competence ("scope") of total agents, that is, agents with a communication capability as a total function in mathematical sense, cannot be strictly improved.

This paper is structured as follows. Below are some general terminology and notation that will be used. Contextual coordination of [Ago01a] is summarized in Section 3, even if the protocol we present in this paper allows the agents to communicate their "guesses." (Recall that this was not the case in [Ago01a].) An example of coordination among sets of "self-centered, learning agent" and the comparison of contextual coordination with and without explicit guesses (cf. [Ago01a]) are also advanced. The basic components of "absolute" coordination but the criterion of coordination success are introduced in Section 4, and completed in Section 5 with six variants of success criteria. The resulting paradigms are discussed and compared in detail. Sections 6 and 7 report on the behavior of "basic agents," with special reference to two kinds of "forgiving" and "blind" agents, and the coordination competence thereof. We conclude the paper in Section 8, just after some related work has been presented.

Remark. Despite we present more than a glimpse on coordination and some sample results, this article should also be read as an attempt to further address the work in inductive coordination—say [Ago01b, AM01] and the references cited there, into the rapidly emerging field of Knowledge Management and Discovery; see for instance [CJLZ99, FPSSU96, FPSS96, BA96, BBM00].

2 Preliminaries

2.1 Sequences

We denote the set $\{0, 1, 2, ...\}$ of natural numbers by N. We denote the usual linearly ordered structure with domain N by ω. Let η be an infinite sequence.[1] For $i \in N$, we write $\eta|_i$ for the proper initial sequence of length i in η. For every finite or infinite sequence ζ of length $n > k$, $k \in N$, we let $\zeta[k]$ denote the finite sequence $\langle \zeta_0 \cdots \zeta_k \rangle$ and ${}_k|\zeta$ denote the sequence obtained from ζ by deleting its first $k+1$ elements. We write $|\sigma|$ for the length of a finite sequence, and $\emptyset$ for the finite sequence of length zero, Thus, $\zeta[k] = \zeta|_{k+1}$, and ${}_0|\zeta = \emptyset$ if $|\zeta| = 1$. We write σ_i or also $(\sigma)_i$ for the ith element of σ, $0 \leq i < |\sigma|$, and $last(\sigma)$ for the last element in σ. The set of elements in a (finite, infinite) sequence τ is denoted by $range(\tau)$.

2.2 A First-Order Framework

We write $\mathcal{L}_{form}$ to denote a first-order language with equality built up from a (countable, decidable) vocabulary $\mathcal{L}$ (equality symbol is: $\doteq$). Language $\mathcal{L}_{form}$ contains a countably infinite set $Var = \{v_i \mid i \in N\}$ of variables.The members of

[1]By "infinite sequence" we shall always mean an ω-sequence, or a total function defined on N.

$\mathcal{L}_{form}$ are called **formulas**. To avoid ambiguities, we let formulas contain parentheses as auxiliary signs. We write $\mathcal{L}_{sen}$ to denote the set of **sentences**, that is, the subset of $\mathcal{L}_{form}$ containing no free variables. We write $\mathcal{L}_{basic}$ to denote the set of literals of $\mathcal{L}_{form}$. The members of $\mathcal{L}_{basic}$ are called **basic formulas**.

Our semantic notions are standard. Let $\mathcal{S}$ be a structure that interprets $\mathcal{L}$. Let $\models$ denote the model theoretic concept of truth in a structure. Then $\mathcal{S}$ is a **model** of $\Gamma \subseteq \mathcal{L}_{form}$, and Γ is said to be **satisfiable in** $\mathcal{S}$, if there is an assignment $h : Var \longrightarrow \text{dom}(S)$ with $\mathcal{S} \models \Gamma[h]$. Γ is **satisfiable** if it is satisfiable in some structure. An assignment h to $\mathcal{S}$ is **complete** if h is a mapping onto $\text{dom}(S)$. The **basic diagram** of $\mathcal{S}$ **under** complete assignment h is the subset of $\mathcal{L}_{basic}$ made true in $\mathcal{S}$ via h. The class of models of Γ is denoted by $MOD(\Gamma)$.

2.3 Environments and Contexts

Let *SEQ* denote the collection of all finite sequences over the set $\mathcal{L}_{basic}$ of atomic formulas on vocabulary $\mathcal{L}$ and their negations. We define an **environment** to be any infinite sequence over $\mathcal{L}_{basic}$. Thus, for all $\sigma \in SEQ$, there is an environment e such that $\sigma = e|_n$ with $n = |\sigma|$. In this technical sense, *SEQ* then denotes the collection of all proper initial sequences of any environment. To consider consistent data-streams, we need to relate them to a structure. We do this in the next definition.

(1) DEFINITION: Let structure $\mathcal{S}$ and complete assignment h to $\mathcal{S}$ be given. An environment e is **for** $\mathcal{S}$ **via** h just in case $range(e) = \{\beta \in \mathcal{L}_{basic} \mid \mathcal{S} \models \beta[h]\}$. An environment e is **for** $\mathcal{S}$ just in case e is an environment for $\mathcal{S}$ via some complete assignment.

In other words, an environment for a structure $\mathcal{S}$ via complete assignment h lists the basic diagram of $\mathcal{S}$ under h.

(2) *Remark:* Many alternative definitions of an "environment" are possible: incomplete, noisy, imperfect, recursive, sential, multi-agent, etcetera. "Real" environments may suffer omissions (incompleteness), erroneous intrusions (noise), or both omissions and intrusions.

(3) DEFINITION: A **context** is any subset of $\mathcal{L}_{sen}$.

From now to the end of this paper we denote a context: π.

3 Contextual Coordination

A problem which has been left unexplored in [Ago01a] is how an agent's guesses in a given context about a problem to solve may influence the agents choices. That problem was there addressed "as the last remark on cooperation." In this paper we present a paradigm of coordination where the agents' guesses are not hidden to any other agent concerned. So, we discuss and compare a variant of

π-coordination that provides protocols for explicit guesses. The new paradigm, however, is equivalent to that of [Ago01a], as we will see in subsection 3.2 below.

3.1 Explicit Guesses

Let π be a context. The paradigm of π-coordination by [Ago01a, Sec. 3] concerns agents whose input's domain is defined on sequences of basic formulas only; the sentences in π are therefore not in the agents' input. As a consequence, if a learning agent wants to communicate his guesses to an agent, he must use an indirect communication "protocol". For example, the agent could use a coded message by Gödel numbering together with the following strategy: at each even stage of the interaction sequence, guess "$v_n \doteq v_n$" by the external ability and sentence θ by the internal ability, where n is the Gödel number of θ; at each odd stage of the interaction sequence, act and guess as context situation requires.[2] By following this strategy, in this section we show that an agent who communicates his guesses to the opponent at each step of the interaction sequence may be simulated by a learning agent that does not manage explicit guesses. On the other hand, it is quite clear that any agent who "hides" his guesses—this is the case of learning agents, may be simulated by an agent that always shows his own guesses. This can be done simply by requiring the agent to ignore in input the guesses of the opponent. It follows that the two paradigms with and without explicit guesses are equivalent. However, the paradigm with no explicit guess is essentially simpler, so that our choice of a paradigm of coordination without explicit guesses as more fundamental is justified in this way.

In the rest of this section we prepare ourself to state the equivalence of the paradigms with and without explicit guesses formally. We begin with the next definition.

(4) Definition: Let *SEN* denote the collection of all the *finite* sequences over $\mathcal{L}_{sen}$. A **learning agent with explicit guess** is a pair $\langle \Psi , \mathbf{A} \rangle$, where Ψ is any mapping from $SEQ \times SEN$ to $\mathcal{L}_{basic} \times \mathcal{L}_{sen}$, and $\mathbf{A}$ is a nonempty (countable) class of structures.

Note that for fixed $\tau \in SEN$, $\langle \lambda\sigma.\Psi(\sigma, \tau) , \mathbf{A} \rangle$ is a learning agent. Similarly to learning agents in [Ago01a], of the two components of any learning agent with explicit guess, the first is called **communication function**, or "ability," while the second component is called **background world**. We say that $\langle \Psi , \mathbf{A} \rangle$ (or also Ψ) is **based on** $\mathbf{A}$, Ψ is **of** $\langle \Psi , \mathbf{A} \rangle$, and $\langle \Psi , \mathbf{A} \rangle$ **has** Ψ. If Ψ is total on some $I \subseteq SEQ \times SEN$, we say that Ψ is a **strategy in** I. We say that Ψ is a **strategy** if Ψ is a strategy in $SEQ \times SEN$. We write $\mathbf{A}_\Psi$ for $\mathbf{A}$ and $\Psi_\mathbf{A}$ for Ψ (or also $\langle \Psi , \mathbf{A} \rangle$) just in case Ψ is based on $\mathbf{A}$. We write Λ^{lg} for the class of all learning agents with explicit guess.

Let learning agents $\langle \Psi , \mathbf{A} \rangle$, $\langle \Phi , \mathbf{B} \rangle$ with explicit guess be given. The **interac-**

[2] For background on arithmetization and Gödel numbers for an arbitrary first-order theory see for instance [Men87, Ch. 3].

tion sequence of $\langle \Psi , \mathbf{A} \rangle$ and $\langle \Phi , \mathbf{B} \rangle$ is the infinite sequence

$$D^g_{\Psi,\Phi} = (\langle \overline{\Psi}_i , \overline{\Phi}_i \rangle \, : \, i \in N),$$

where $\overline{\Psi}_i$ is the ith move of Ψ and $\overline{\Phi}_i$ is the ith move of Φ, defined by induction as follows.

i. $\overline{\Psi}_{00} = (\Psi(\emptyset, \emptyset))_0$ and $\overline{\Psi}_{10} = (\Psi(\emptyset, \emptyset))_1$;
$\overline{\Phi}_{00} = (\Phi(\emptyset, \emptyset))_0$ and $\overline{\Phi}_{10} = (\Phi(\emptyset, \emptyset))_1$.

ii. $\overline{\Psi}_{0n+1} = (\Psi(\overline{\Phi}_0[n], \overline{\Phi}_1[n]))_0$ and $\overline{\Psi}_{1n+1} = (\Psi(\overline{\Phi}_0[n], \overline{\Phi}_1[n]))_1$;
$\overline{\Phi}_{0n+1} = (\Phi(\overline{\Psi}_0[n], \overline{\Psi}_1[n]))_0$ and $\overline{\Phi}_{1n+1} = (\Phi(\overline{\Psi}_0[n], \overline{\Psi}_1[n]))_1$.

Let $k \in N$ be given. The **interaction sequence** of $\langle \Psi , \mathbf{A} \rangle$ and $\langle \Phi , \mathbf{B} \rangle$ **starting at** k is the infinite sequence

$${}^kD^s_{\Psi,\Phi} = (\langle \, {}_k|\overline{\Psi}_i \, , {}_k|\overline{\Phi}_i \, \rangle \, : \, i \in N),$$

where ${}_k|\overline{\Psi}_i$ is the ith move of Ψ **after** k and ${}_k|\overline{\Phi}_i$ is the ith move of Φ **after** k. The **response sequence**

$$R(\Psi, \Phi) = (\overline{\Psi}_i \, : \, i \in N)$$

is the (finite or infinite) sequence of moves by learning agent with explicit guess and ability Ψ in response to learning agent with explicit guess and ability Φ. The **response sequence** $R(\Phi, \Psi) =(\overline{\Phi}_i \, : \, i \in N)$ is the same sequence with the roles of Ψ and Ψ reversed.

Now we are ready to define coordination "success" among learning agents with explicit guess. The next definition thus extends Definition (4) of [Ago01a] to the present framework.

(5) Definition: Let $\pi \subseteq \mathcal{L}_{sen}$ and learning agents $\langle \Psi , \mathbf{A} \rangle$, $\langle \Phi , \mathbf{B} \rangle$ with explicit guess be given. We say that $\langle \Psi , \mathbf{A} \rangle$ π**-coordinates with** $\langle \Phi , \mathbf{B} \rangle$ **by explicit guesses** (written: $\Psi \rightleftharpoons_{\pi^g} \Phi$) just in case for some $s, t \in N$, there is $\theta \in \pi$ such that:

- (a) ${}_s|\overline{\Psi}_0$ is an environment for some $\mathcal{A} \in \mathbf{A}$;
- (b) ${}_t|\overline{\Phi}_0$ is an environment for some $\mathcal{B} \in \mathbf{B}$
- (c) for all but finitely many $n \in N$, $({}_s|\overline{\Psi}_1)_n = ({}_t|\overline{\Phi}_1)_n = \theta$;
- (d) $\mathcal{A} \models \theta$ and $\mathcal{B} \models \theta$.

A **paradigm of** π**-coordination and explicit guesses** is a model coordination where "agents" are learning agents with explicit guess, "dynamics" are interaction sequences as defined above, and "coordination success" is that of Definition (5).

Follows an example of π^g-coordination. The example concerns a special set of "self-centered learning agent" with explicit guess. We rely on the following definition, to state which some further terminology is needed.

(6) DEFINITION: Let $\sigma \in SEQ$, $\sigma' \in SEN$ and learning agent with explicit guess $\langle \Psi \,, \mathbf{A} \rangle$ be given. We define the **communication sequence** $\overline{(\Psi(\sigma, \sigma'))_0} \in SEQ$ by induction on the length of σ and σ' as follows. $\overline{(\Psi(\emptyset, \emptyset))_0} = (\Psi(\emptyset, \emptyset))_0$. Suppose that $\overline{(\Psi(\tau, \eta))_0}$ is defined for $\tau \in SEQ$ and $\eta \in SEN$. Given $\beta \in \mathcal{L}_{basic}$ and $\theta \in \mathcal{L}_{sen}$, define $\overline{(\Psi(\tau\beta, \eta\theta))_0} = \overline{(\Psi(\tau, \eta))_0}^\frown (\Psi(\tau\beta, \eta\theta))_0$.

Note that when $\overline{(\Psi(\sigma, \sigma'))_0}$ is defined, $|\overline{(\Psi(\sigma, \sigma'))_0}| > 0$.

Let $SEQ_\Psi = \{\overline{(\Psi(\sigma, \sigma'))_0} \mid \sigma \in SEQ, \sigma' \in \mathcal{L}_{sen}\}$ denote the collection of all the finite sequences produced on any input by a learning agent with explicit guess and ability Ψ. Then:

(7) DEFINITION: Let $\pi \subseteq \mathcal{L}_{sen}$ be given. We say that learning agent with explicit guess $\langle \Psi \,, \mathbf{A} \rangle$ is $self^\pi$**-centered** just in case for all strategies Φ in SEQ_Ψ, there are $\mathcal{A} \in \mathbf{A}$ and $\theta \in \pi$ such that:

- (a) $R(\Psi_0, \Phi)$ is an environment for $\mathcal{A}$;
- (b) for all but finitely many $n \in N$, $(R(\Psi_1, \Phi))_n = \theta$;
- (c) $\mathcal{A} \models \theta$.

In other words, a $self^\pi$-centered learning agent with explicit guess enumerates with any other learning agents an environment for some structure $\mathcal{S}$ in the background world the agent is based on, and guesses with the same agent a sentence true in $\mathcal{S}$.

(8) PROPOSITION: [Ago01b] Let $\pi \subseteq \mathcal{L}_{sen}$. For every nonempty set Σ of $self^\pi$-centered learning agents with explicit guess, there is a learning agent with explicit guess who π^g-coordinates with each member of Σ.

Proof: We follow [AM01]. Let Σ be a nonempty set of $self^\pi$-centered learning agents with explicit guess. We define a learning agent $\langle \Psi \,, \mathbf{A} \rangle$ with explicit guess such that:

- (a) $\mathbf{A}$ is the set of structures $\mathcal{S}$ such that there are $\langle \Phi \,, \mathbf{B} \rangle$, $\langle \Phi' \,, \mathbf{B}' \rangle \in \Sigma$ and $\theta \in \pi$ with $\mathcal{S} \models \theta$ such that $R(\Phi_0, \Phi')$ is an environment for $\mathcal{S}$ and for all $n \in N$, $(R(\Phi_1, \Phi'))_n = \theta$. Observe that $\mathbf{A}$ is nonempty.
- (b) $\Psi(\emptyset, \emptyset) = \langle v_0 \doteq v_0 \,, \theta' \rangle$, for some $\theta' \in \pi$ such that $\mathcal{A} \models \theta'$ with $\mathcal{A} \in \mathbf{A}$. For all $\sigma \in SEQ$ and all $\sigma' \in SEN$ with $|\sigma| = |\sigma'|$, if $|\sigma| > 0$ then $\Psi(\sigma, \sigma') = \langle \sigma_{|\sigma|-1} \,, \sigma'_{|\sigma|-1} \rangle$.

[Thus, when playing with learning agents $\langle \Phi \,, \mathbf{B} \rangle \in \Sigma$, Ψ starts by moving "safe" and then copies Φ's last move one step later.] It follows that

$$R(\Psi_0, \Phi) = v_0 \doteq v_0{}^\frown R(\Phi_0, \Psi)$$

and

$$R(\Psi_1, \Phi) = \forall v_0 \, (v_0 \doteq v_0)^\frown R(\Phi_1, \Psi).$$

Since $\langle \Phi \,, \mathbf{B} \rangle$ is $self^\pi$-centered, $R(\Psi_0, \Phi)$ is an environment for some structure $\mathcal{A} \in \mathbf{A}$ and there is $\theta \in \pi$ such that for all $n \in N$, $(R(\Psi_1, \Phi))_n = \theta$ with $\mathcal{A} \models \theta$.

From the definition of Σ it follows that $\mathcal{A} \in \mathbf{A}$. So, it is easy to verify that $\langle \Psi , \mathbf{A} \rangle$ π^g-coordinates with $\langle \Phi , \mathbf{B} \rangle$. ■

3.2 Comparison

We now wish to compare contextual coordination with and without explicit guesses, the latter being defined in [Ago01a]. The folowing definition is useful.

(9) DEFINITION: Let $\pi \subseteq \mathcal{L}_{sen}$ and nonempty classes $\mathbf{A}$ and $\mathbf{B}$ of structures be given. We say that $\mathbf{A}$ **π^g-matches B** just in case there are learning agents with explicit guess $\langle \Psi , \mathbf{A} \rangle$ and $\langle \Phi , \mathbf{B} \rangle$ such that $\langle \Psi , \mathbf{A} \rangle$ π^g-coordinates with $\langle \Phi , \mathbf{B} \rangle$.

We then write "$\pi^g \overset{\triangleleft}{\approx} \pi$" just in case for whatever vocabulary $\mathcal{L}$ and for all nonempty classes $\mathbf{A}$, $\mathbf{B}$ of $\mathcal{L}$-structures, if $\mathbf{A}$ π^g-matches $\mathbf{B}$ then $\mathbf{A}$ π-matches $\mathbf{B}$. Let $\approx$ and $\triangleleft$ be the symmetric and asymmetric components of $\overset{\triangleleft}{\approx}$ respectively.

The next proposition gives the exact sense by which a learning agent with explicit guess can be simulated by a learning agent (with no explicit guess), and vice versa.

Let Λ^l denote the class of all learning agents. (Recall that Λ^{lg} is the class of learning agents with explicit guess.)

(10) PROPOSITION: [AM01, Ago01b] Learning agents with explicit guess can be simulated by learning agents (without explicit guess) and vice versa in the following sense.

(a) There is an uniform map Γ from Λ^{lg} to Λ^l such that for all learning agents with explicit guess $\langle \Psi , \mathbf{A} \rangle$, $\langle \Phi , \mathbf{B} \rangle$ and for all $\pi \subseteq \mathcal{L}_{sen}$, $\langle \Psi , \mathbf{A} \rangle$ π^g-coordinates with $\langle \Phi , \mathbf{B} \rangle$ iff $\Gamma(\langle \Psi , \mathbf{A} \rangle)$ π-coordinates with $\Gamma(\langle \Phi , \mathbf{B} \rangle)$.

(b) There is an uniform map Γ' from Λ^l to Λ^{lg} such that for all learning agents $\langle \Psi , \mathbf{A} \rangle$, $\langle \Phi , \mathbf{B} \rangle$ and for all $\pi \subseteq \mathcal{L}_{sen}$, $\langle \Psi , \mathbf{A} \rangle$ π-coordinates with $\langle \Phi , \mathbf{B} \rangle$ iff $\Gamma'(\langle \Psi , \mathbf{A} \rangle)$ π^g-coordinates with $\Gamma'(\langle \Phi , \mathbf{B} \rangle)$.

Proof: We prove (a). Let $\sigma \in SEQ$, $i \in N^+$, and learning agents with explicit guess $\langle \Psi , \mathbf{A} \rangle$ and $\langle \Phi , \mathbf{B} \rangle$ be given. We define $\Psi'(\sigma) \in \mathcal{L}_{basic} \times \mathcal{L}_{sen}$ from Ψ and $\Phi'(\sigma) \in \mathcal{L}_{basic} \times \mathcal{L}_{sen}$ from Φ as follows.

(a) $\Psi'(\emptyset) = \Psi(\emptyset, \emptyset)$.

(b) If $|\sigma| = 2i$, let for $j = 0, 1, \ldots, i-1$ $\theta_j \in \mathcal{L}_{sen}$ be such that $\ulcorner \theta_j \urcorner = n$ [*i.e.* the Gödel number of θ_j] if $\sigma_{2j} = (v_n \doteq v_n)$, and let θ_j be any $\mathcal{L}$-sentence otherwise. Then $(\Psi'(\sigma))_0 = (\Psi(\langle \sigma_1 \sigma_3 \cdots \sigma_{2i-3} \rangle, \langle \theta_0 \cdots \theta_{i-1} \rangle))_0$.

(c) If $|\sigma| = 2i+1$ then $(\Psi'(\sigma))_1 = (\Psi(\langle \sigma_1 \sigma_3 \cdots \sigma_{2i-1} \rangle, \langle \theta_0 \cdots \theta_i \rangle))_1$.

(d) If $\sigma_{2j} \neq (v_n \doteq v_n)$ for all $n \in N$, $\Psi'(\sigma)$ is arbitrary.

In similar way we define Φ'. It follows from the definitions of Ψ' and Φ' that $(R(\Psi_0, \Phi))_i = (R(\Psi'_0, \Phi'))_{2i}$ and that $(R(\Psi_1, \Phi))_i = (R(\Psi'_1, \Phi'))_{2i+1}$.

Define $\Gamma(\langle \Psi , \mathbf{A}\rangle) = \langle \Psi' , \mathbf{A}\rangle$ and $\Gamma(\langle \Phi , \mathbf{B}\rangle) = \langle \Phi' , \mathbf{B}\rangle$. Hence $\langle \Psi , \mathbf{A}\rangle$ π^g-coordinates with $\langle \Phi , \mathbf{B}\rangle$ iff $\Gamma(\langle \Psi , \mathbf{A}\rangle)$ π-coordinates with $\Gamma(\langle \Phi , \mathbf{B}\rangle)$.

We prove (b). Let learning agents (without explicit guess) $\langle \Psi , \mathbf{A}\rangle$ and $\langle \Phi , \mathbf{B}\rangle$ be given. For all $\sigma \in \mathit{SEQ}$ and all $\sigma' \in \mathit{SEN}$, we define $\Psi'(\sigma, \sigma') = \Psi(\sigma)$ and $\Phi'(\sigma, \sigma') = \Phi(\sigma)$. [Thus Ψ' and Φ' ignore the explicit guess and act as Ψ and Φ, respectively.] Let $\Gamma'(\langle \Psi , \mathbf{A}\rangle) = \langle \Psi' , \mathbf{A}\rangle$ and $\Gamma'(\langle \Phi , \mathbf{B}\rangle) = \langle \Phi' , \mathbf{B}\rangle$. Clearly $\langle \Psi , \mathbf{A}\rangle$ π-coordinates with $\langle \Phi , \mathbf{B}\rangle$ iff $\Gamma'(\langle \Psi , \mathbf{A}\rangle)$ π^g-coordinates with $\Gamma'(\langle \Phi , \mathbf{B}\rangle)$. ■

Before returning to technical issues, we indulge in a general remark on coordination. The paradigms of contextual coordination (with or without explicit guesses) are designed to examine the logic of long-term interaction. They capture the idea that an agent will take into account the effect of his or her current behavior on the other agent's future behavior, and aim to explain phenomena like cooperation and threats.

4 Absolute Coordination

Recall that in this paper we restrict attention to dynamics based on simultaneous moves, that is, the agents make decisions at the same time, and to pairwise interaction, that is, interactions involve only two agents at time. Simultaneous moves and pairwise interaction are common features of contextual and absolute coordination.

4.1 Components

The basic components of "absolute" coordination but the criterion of coordination success are now to be introduced in detail.

4.1.1 Agents.

An agent in an absolute coordination paradigm, or **basic agent**, is a pair $\langle \Psi , \mathbf{A}\rangle$, where Ψ is any mapping of SEQ into $\mathcal{L}_{basic}$, and $\mathbf{A}$ is a nonempty class of (countable) structures. Intuitively, faced with $\sigma \in \mathit{SEQ}$, a basic agent $\langle \Psi , \mathbf{A}\rangle$ believes **action** $\Psi(\sigma)$. Since class $\mathbf{A}$ is interpreted as to represent the agent's "preferences and beliefs," $\Psi(\sigma)$ is expected to be true in some structure in $\mathbf{A}$.[3] For basic agent $\langle \Psi , \mathbf{A}\rangle$, the first component Ψ is called **communication function** or "ability". The second component $\mathbf{A}$ is called **background world**. We say that $\langle \Psi , \mathbf{A}\rangle$ (or also Ψ) is **based on** $\mathbf{A}$, Ψ is **of** $\langle \Psi , \mathbf{A}\rangle$, and $\langle \Psi , \mathbf{A}\rangle$ **has** Ψ. We write $\mathbf{A}_\Psi$ for $\mathbf{A}$ and $\Psi_\mathbf{A}$ for Ψ (or also $\langle \Psi , \mathbf{A}\rangle$) just in case Ψ is based on $\mathbf{A}$. We refer to $\mathcal{L}$ as the agent's vocabulary, and to $\mathcal{L}_{basic}$ as the agent's language. We write Λ^b for the class of all basic agents and $\Lambda^b(\mathbf{A})$ for the class of all basic agents based on $\mathbf{A}$.

[3]This is not a strict requirement, however, but the underlying intuition should help the reader in understanding the criterion of success of any slow-full coordination paradigm. By the same interpretation, we require $\mathbf{A}$ to be nonempty, so that agents are assumed to believe something.

4.1.2 Dynamics.

We consider formally the interaction, or "protocol," between two basic agents.

(11) DEFINITION: Let basic agents $\langle \Psi , \mathbf{A} \rangle$ and $\langle \Phi , \mathbf{B} \rangle$ be given.

(a) The **interaction sequence** (or "play") of $\langle \Psi , \mathbf{A} \rangle$ and $\langle \Phi , \mathbf{B} \rangle$ is the infinite sequence

$$D_{\Psi,\Phi} = (\langle \overline{\Psi}_i , \overline{\Phi}_i \rangle \; : \; i \in N),$$

where $\overline{\Psi}_i$ is the ith move of Ψ and $\overline{\Phi}_i$ is the ith move of Φ, defined by induction as follows.

i. $\overline{\Psi}_0 = \Psi(\emptyset)$ and $\overline{\Phi}_0 = \Phi(\emptyset)$.
ii. $\overline{\Psi}_{n+1} = \Psi(\overline{\Phi}[n])$ and $\overline{\Phi}_{n+1} = \Phi(\overline{\Psi}[n])$.

(b) Let $k \in N$ be given. The **interaction sequence** of $\langle \Psi , \mathbf{A} \rangle$ and $\langle \Phi , \mathbf{B} \rangle$ **starting at** k is the infinite sequence

$${}^{k}D_{\Psi,\Phi} = (\langle {}_{k}|\overline{\Psi}_i , {}_{k}|\overline{\Phi}_i \rangle \; : \; i \in N),$$

where ${}_{k}|\overline{\Psi}_i$ is the ith element in ${}_{k}|\overline{\Psi}$ and ${}_{k}|\overline{\Phi}_i$ is the ith element in ${}_{k}|\overline{\Phi}$.

We then say that ${}_{k}|\overline{\Psi}_i$ is the ith move of Ψ starting at k and ${}_{k}|\overline{\Phi}_i$ is the ith move of Φ starting at k. Observe that $D_{\Psi,\Phi} = \langle \overline{\Psi}_0 , \overline{\Phi}_0 \rangle {}^{0}D_{\Psi,\Phi}$.

(12) *Remark:* The definition of interaction sequence ${}^{k}D_{\Psi,\Phi}$ depends only on the agents' abilities Ψ and Φ; no background worlds are involved. We shall see later in this section that basic agents' background worlds are relevant to determine the criterion of coordination success.

The **response sequence**

$$R(\Psi, \Phi) = (\overline{\Psi}_i \; : \; i \in N)$$

is the (finite or infinite) sequence of moves by basic agent Ψ in response to basic agent Φ, and the **response sequence**

$$R(\Phi, \Psi) = (\overline{\Phi}_i \; : \; i \in N)$$

is the sequence of moves by Φ in response to Ψ. Again, notice that $R(\Psi, \Phi)$ is finite if and only if at any interaction step $i \in N$, $\Psi(\overline{\Psi}|_i)$ or $\Psi(\overline{\Phi}|_i)$ is undefined. If this is the case, it is immediate to verify that $R(\Phi, \Psi)$ is finite also. Consistently with the notation adopted on infinite sequences within the 01-coordination paradigm, ${}_{k}|R(\Psi, \Phi)|_n$ denotes the finite initial sequence in ${}_{k}|R(\Psi, \Phi)$ of length n, and ${}_{k}|R(\Psi, \Phi)_n$, or also $({}_{k}|R(\Psi, \Phi))_n$ denotes the nth element of ${}_{k}|R(\Psi, \Phi)$. Thus, ${}_{k}|R(\Psi, \Phi)|_{n+1} = {}_{k}|R(\Psi, \Phi)|_n \overline{\Psi}_n$ and $R(\Psi, \Phi) = \overline{\Psi}_0 {}^\frown {}_{0}|R(\Psi, \Phi)$.

Agents and dynamics combine well in the following definition, that will also be useful elsewhere.

(13) DEFINITION: Let $\pi \subseteq \mathcal{L}_{sen}$ be given. We say that basic agent $\langle \Psi, \mathbf{A} \rangle$ is $self^{\pi}$**-centered** just in case for every $\Phi : SEQ \rightarrow \mathcal{L}_{basic}$, there is $\mathcal{A} \in \mathbf{A}$ such that $R(\Psi, \Phi)$ is an environment for $\mathcal{A}$.

In other words, a $self^{\pi}$-centered basic agent enumerates with some other basic agent an environment for some structure in the background world the agent is based on.

5 Some Alternative Paradigms

Even if restricting the study of coordination to a first-order setting—as we do, the variety of interesting paradigms is clearly huge. An argument supporting our choices below is that "binary coordination" [MO99] may be embedded as special cases in our setting. In short, binary coordination is a pairwise, limiting process, where each of two agents chooses his or her actions among two possible choices, conventionally denoted by 0, 1. In particular, the agents' "beliefs, preferences, and intentions" are not captured by Montagna and Osherson's model. (We refer the interested reader to [AdJM01, Ago01b] for further discussion.)

In what follows, we describe a glimpse of the vast array of paradigms of coordination that can be defined. Our presentation proceeds by modifying the interpretation of "success." We do this in the next subsection. Successively, we compare and discuss the criteria of success proposed.

5.1 Six Criteria of Success

We distinguish six variants of coordination success criteria. Each stipulates the conditions under which two basic agents can be said to coordinate. All our variants are based on the fundamental use of the agents' background world or "preferences and beliefs set." In what follows, w, l, f and s may be read as "weak," "local," "full," "slow," respectively.

One of our more liberal variants relies on the following generalization of Definition (1). (Recall that if $\mathcal{A}$ and $\mathcal{B}$ are $\mathcal{L}$-structures with $\mathrm{dom}(\mathcal{A}) \subseteq \mathrm{dom}(\mathcal{B})$ and the inclusion map $i : \mathrm{dom}(\mathcal{A}) \longrightarrow \mathrm{dom}(\mathcal{B})$ is an embedding, then $\mathcal{B}$ is said to be an **extension of** $\mathcal{A}$, or also that $\mathcal{A}$ is a **substructure of** $\mathcal{B}$; see for instance [Hod97] for background.)

(14) DEFINITION: Let structure $\mathcal{S}$ be given. An environment e is **partial for** $\mathcal{S}$ just in case there are a substructure $\mathcal{T}$ of $\mathcal{S}$ and complete assignment h to $\mathcal{T}$ such that $range(e) = \{\beta \in \mathcal{L}_{basic} \mid \mathcal{T} \models \beta[h]\}$.

Thus, a partial environment for $\mathcal{S}$ is an environment for $\mathcal{S}$ in the special case $\mathcal{S} = \mathcal{T}$. Also note that a partial environment for $\mathcal{S}$ is an environment for some $\mathcal{T}$.

(15) DEFINITION: Let basic agents $\langle \Psi, \mathbf{A} \rangle$ and $\langle \Phi, \mathbf{B} \rangle$ be given. We say that:

(a) $\langle\Psi\,,\mathbf{A}\rangle$ sf°**-coordinates with** $\langle\Phi\,,\mathbf{B}\rangle$ just in case for some $s,t\in N$, ${}_s|\overline{\Psi}$ is an environment for some $\mathcal{A}\in\mathbf{A}$, ${}_t|\overline{\Phi}$ is an environment for some $\mathcal{B}\in\mathbf{B}$, and for all $n\in N$, ${}_s|\overline{\Psi}|_n$ is satisfiable in $\mathcal{B}$ and ${}_t|\overline{\Phi}|_n$ is satisfiable in $\mathcal{A}$.

(b) $\langle\Psi\,,\mathbf{A}\rangle$ f**-coordinates with** $\langle\Phi\,,\mathbf{B}\rangle$ just in case $\overline{\Psi}$ is an environment for some $\mathcal{A}\in\mathbf{A}$, $\overline{\Phi}$ is an environment for some $\mathcal{B}\in\mathbf{B}$, and for all $n\in N$, $\overline{\Psi}|_n$ is satisfiable in $\mathcal{B}$ and $\overline{\Phi}|_n$ is satisfiable in $\mathcal{A}$.

(c) $\langle\Psi\,,\mathbf{A}\rangle$ lf**-coordinates with** $\langle\Phi\,,\mathbf{B}\rangle$ just in case $\overline{\Psi}$ and $\overline{\Phi}$ are environments for some structure $\mathcal{S}$, and for all $n\in N$, $\overline{\Psi}|_n$ is satisfiable in some $\mathcal{A}\in\mathbf{A}$ and $\overline{\Phi}|_n$ is satisfiable in some $\mathcal{B}\in\mathbf{B}$.

(d) $\langle\Psi\,,\mathbf{A}\rangle$ $l\,s$**-coordinates with** $\langle\Phi\,,\mathbf{B}\rangle$ just in case for some $s,t\in N$, ${}_s|\overline{\Psi}$ and ${}_t|\overline{\Phi}$ are environments for some structure $\mathcal{S}$, and for all $n\in N$, ${}_s|\overline{\Psi}|_n$ is satisfiable in some $\mathcal{A}\in\mathbf{A}$ and ${}_t|\overline{\Phi}|_n$ is satisfiable in some $\mathcal{B}\in\mathbf{B}$.

(e) $\langle\Psi\,,\mathbf{A}\rangle$ $l\,s^\circ$**-coordinates with** $\langle\Phi\,,\mathbf{B}\rangle$ just in case for some $s,t\in N$, ${}_s|\overline{\Psi}$ and ${}_t|\overline{\Phi}$ are environments for some structure $\mathcal{S}$, and for some $\mathcal{A}\in\mathbf{A}$ and some $\mathcal{B}\in\mathbf{B}$, ${}_s|\overline{\Psi}|_n$ is satisfiable in $\mathcal{A}$ and ${}_t|\overline{\Phi}|_n$ is satisfiable in $\mathcal{B}$ for all $n\in N$.

(f) $\langle\Psi\,,\mathbf{A}\rangle$ w**-coordinates with** $\langle\Phi\,,\mathbf{B}\rangle$ just in case for some $s,t\in N$, ${}_s|\overline{\Psi}$ is a partial environment for some $\mathcal{A}\in\mathbf{A}$, ${}_t|\overline{\Phi}$ is a partial environment for some $\mathcal{B}\in\mathbf{B}$, and for all $n\in N$, ${}_s|\overline{\Psi}|_n$ is satisfiable in $\mathcal{B}$ and ${}_t|\overline{\Phi}|_n$ is satisfiable in $\mathcal{A}$.

Note that structures $\mathcal{A}$, $\mathcal{B}$ in clauses (c) and (d) depend on n, while $\mathcal{A}$, $\mathcal{B}$ do not depend on n in all other cases. Moreover, the environments mentioned in clauses (c), (d) and (e) are for the *same* structure $\mathcal{S}$, although each of such environments can be for $\mathcal{S}$ via a different complete assignment. In the cases (c), (d) and (e) of the definition, we also say that $\langle\Psi\,,\mathbf{A}\rangle$ and $\langle\Phi\,,\mathbf{B}\rangle$ lf- ($l\,s$-, $l\,s^\circ$-) coordinate **on** $\mathcal{S}$. We sometimes write $lf(\mathcal{S})-$ ($l\,s(\mathcal{S})-$, $l\,s^\circ(\mathcal{S})-$) in place of lf- ($l\,s$-, $l\,s^\circ$-).

Let X be any of the prefixes:

(16) $sf^\circ \quad f \quad lf \quad l\,s \quad l\,s^\circ \quad w.$

We then write $\Psi \rightleftharpoons_X \Phi$ to shorten "$\langle\Psi\,,\mathbf{A}\rangle$ X-coordinates with $\langle\Phi\,,\mathbf{B}\rangle$."

Observe:

(17) LEMMA: For all basic agents $\langle\Psi\,,\mathbf{A}\rangle$, $\langle\Phi\,,\mathbf{B}\rangle$,

(a) if $\langle\Psi\,,\mathbf{A}\rangle$ is weakly self-centered and total, then $\Psi \rightleftharpoons_X \Psi$;
(b) if $\Psi \rightleftharpoons_X \Phi$ then $\Phi \rightleftharpoons_X \Psi$.

However X-coordination is not transitive, as is easy to verify by using classes of structures as in Example **??**, so that $\rightleftharpoons_X$ is not an equivalence relation.

5.2 Comments on Criteria

It may be helpful to reflect on the differences between the criteria just introduced.[4]

[4]Most comments in this subsection are dealt systematically in [Ago01b, Sec. 2.5.1].

"Full" coordination (clause (b)) is a criterion of success that models situations where the agents must agree to coordinate from the very beginning up to the limit of the process of coordination. In this sense, one might also say that f-coordination is a "failure-free" criterion of coordination. The f-coordination resulting paradigm separates from the paradigm of sf°-coordination, in the sense that neither does success in the former imply success in the latter nor vice versa. Now, that sf°-coordination is a less stringent requirement of coordination is clear from the definition; think $s \neq t \neq 0$ in clause (a). On the other hand, suppose that basic agent $\langle \Psi , \mathbf{A} \rangle$ f-coordinates with basic agent $\langle \Phi , \mathbf{B} \rangle$. Then Ψ enumerates in the limit an environment $e = R(\Psi, \Phi)$ for some structure $\mathcal{S}$. Suppose that $e_0 \in \mathcal{L}_{basic}$ is neither logically valid nor produced by Ψ further on. Then, from the definition of ${}_{s}|\cdot$ with s be any $s \in N$, ${}_{s}|\overline{\Psi}$ is not an environment for $\mathcal{S}$, even if $s = 0$. (${}_{s}|\overline{\Psi}$ is indeed a partial environment for $\mathcal{S}$.) Hence $\langle \Psi , \mathbf{A} \rangle$ does not sf°-coordinate with $\langle \Phi , \mathbf{B} \rangle$. Similar remarks apply between "local" criteria of clauses (c) and (d). Thus, "local-slow" coordination separates from "local-full" coordination by allowing communication failures: basic agents are not forced to choose their structure at the beginning of the coordination process. In other words, agents are allowed to start the game again from the beginning, although to reach coordination this should occur only finitely often.

"Local" coordination of clauses (c)–(e) is more liberal than the two previous (non-local) criteria only in the sense that there are agents based on fixed classes of structures that coordinate "locally" but such that no agents based on the same classes coordinate in a non-local manner. If success by local coordination is compared to success by non-local coordination for the *same* basic agents, however, then local coordination separates from non-local coordination, in the sense that neither does success in the former imply success in the latter nor vice versa. By local coordination, in fact, basic agents must completely describe the same structure. Since by standard results finite satisfiability does not imply full satisfiability, two basic agents who coordinate non-locally do not necessarily coordinate locally. On the other hand, there are examples of basic agents who coordinate locally but who do not coordinate non-locally. By restricting attention to the most natural 'circled paradigms', we may see that $l\,s^{\circ}$-coordination does not always imply sf°-coordination.

Note that the information on the background world that each basic agent gives to the other basic agent in any local coordination process, that is, the link between the common structure the basic agents end up with in the limit and the agents' own knowledge, is insured by adding the request for each basic agent's output to be finitely consistent with some world in his or her own background world. We can interpret the resulting paradigms as modeling events where *a priori* agreement is made explicit between the agents. The agreement is represented within the paradigms by the structure the agents eventually coordinate on. This structure thus represents in our framework what is usually referred to as *common knowledge*. Similar remarks apply to clauses (b) and (c).

Clearly, every pair of basic agents $l\,s$-coordinate if $l\,s^{\circ}$-coordinate. However, "local slow-full circled" coordination is a less liberal paradigm.[5] To $l\,s^{\circ}$-coordinate, each basic agent has to stabilize to a suitable description of a structure in his or her background world. Each description, moreover, must be consistent with some common structure, which is interpreted as a piece common knowledge (or beliefs) between the two agents. In contrast, to $l\,s$-coordinate, each basic agent eventually enumerates the basic diagram of some common structure, but the range of such enumeration is decomposed into finite sets satisfiable in *different structures at time* of the agent's background world. Thus, $l\,s^{\circ}$-coordination is a strictly stronger requirement.

"Weak" coordination (clause (*f*)) arises if sf°-coordination does. This follows from the definition. The criterion is "weak" in the sense that the information that a basic agent communicates to the other basic agent on the structure (beliefs) he is referring to at the time of action is possibly incomplete. Clearly, sf°-coordination is a strictly stronger criterion than w-coordination, since there are pairs of agents that w-coordinate but that do not sf°-coordinate.

6 Forgiving and Blind Agents

We now wish to investigate two special classes of basic agents so as to provide further insight into X-coordination. In particular, we consider a class of "forgiving" basic agents, who overlook finite parts of information given by their opponents and still coordinate with them. We also consider a class of "blind" basic agents who pay no attention to their opponents' actions and beliefs. We compare these agents and show in turn that every forgiving "self-agent" is blind, but the converse is false. That is, there is a blind, self-centered agent who is not forgiving.

The following two technical definitions will be useful.

(18) DEFINITION: Let $\Phi, \Phi' : SEQ \longrightarrow \mathcal{L}_{basic}$ be given. We say that Φ is a **finite variant of** Φ' just in case for all but finitely many $\sigma \in SEQ$, $\Phi(\sigma) = \Phi'(\sigma)$.

So, two agents having ability Φ and Φ', respectively, are agents whose actions differ on a finite number of data-stream.

The classes of "forgiving" and "blind" that concern us are defined as follows.

(19) DEFINITION: Let X be any of the prefixes in (16). Let basic agent $\langle \Psi , \mathbf{A} \rangle$ be given.

(*a*) $\langle \Psi , \mathbf{A} \rangle$ is **X-forgiving** just in case for all basic agents $\langle \Phi , \mathbf{B} \rangle$, $\langle \Phi' , \mathbf{B} \rangle$ such that Φ is a finite variant of Φ', $\langle \Psi , \mathbf{A} \rangle$ X-coordinates with $\langle \Phi , \mathbf{B} \rangle$ iff $\langle \Psi , \mathbf{A} \rangle$ X-coordinates with $\langle \Phi' , \mathbf{B} \rangle$.

(*b*) $\langle \Psi , \mathbf{A} \rangle$ is **blind** just in case for all total basic agents $\langle \Phi , \mathbf{B} \rangle$, $\langle \Phi' , \mathbf{B}' \rangle$, $R(\Psi, \Phi) = R(\Psi, \Phi')$.

[5] We used a circle in the prefix name to emphasize by a geometric form how $l\,s^{\circ}$-coordination differs from the $l\,s$-coordination. In fact, the use of the structures in each basic agent's background worlds is limited, or "circumscribed," to the structures whose basic diagram is can enumerated by the agents.

Equivalently, $\langle \Psi, \mathbf{A} \rangle$ is blind just in case for all $\sigma, \tau \in SEQ$ with $|\sigma| = |\tau|$, $\Psi(\sigma) = \Psi(\tau)$. We have:

(20) PROPOSITION: [AM01, Ago01b] Every X-forgiving self-centered basic agent is blind.

Proof: By contraposition, suppose basic agent $\langle \Psi, \mathbf{A} \rangle$ is not blind and self-centered. Then there are $\sigma, \sigma' \in SEQ$ such that $|\sigma| = |\sigma'|$ and $\Psi(\sigma) \neq \Psi(\sigma')$. We will show that $\langle \Psi, \mathbf{A} \rangle$ is not X-forgiving by exhibiting a basic agent $\langle \Phi, \mathbf{B} \rangle$ and an ability $\Phi' : SEQ \longrightarrow \mathcal{L}_{basic}$ such that:

- (a) $\langle \Psi, \mathbf{A} \rangle$ X-coordinates with $\langle \Phi, \mathbf{B} \rangle$;
- (b) $\langle \Psi, \mathbf{A} \rangle$ does not X-coordinate with $\langle \Phi', \mathbf{B} \rangle$.

Let define $\mathbf{B} = \mathbf{A}$ and let Φ be defined as follows. We say that $\langle \Phi, \mathbf{B} \rangle$ (or also Φ) **starts with** σ just in case for all $\tau \in SEQ$ with $|\tau| < |\sigma|$, $\Psi(\tau) = (\sigma)_{|\sigma|}$.

- (a) Φ starts with σ;
- (b) for all $\tau \in SEQ$ with $|\tau| = |\sigma|$, $\Phi(\tau) = (v_0 \doteq v_0)$;
- (c) for all $\tau \in SEQ$ with $|\tau| > |\sigma|$, if $\tau_{|\sigma|} = \Psi(\sigma)$ then $\Phi(\tau) = \Psi(\tau|_{|\tau| - |\sigma|})$;
- (d) for all $\tau \in SEQ$ with $|\tau| > |\sigma|$, if $\tau_{|\sigma|} \neq \Psi(\sigma)$ then $\Phi(\tau) = \neg (v_0 \doteq v_0)$.

[In words, Φ starts with σ; then, if there is no previous move by any opponent basic agent to look at, Φ moves "safe". Otherwise, if an opponent basic agent "agrees" with basic agent $\langle \Psi, \mathbf{A} \rangle$ on σ, then Φ starts copying the opponent's moves from the first one. If the opponent basic agent "disagrees" with $\langle \Psi, \mathbf{A} \rangle$ on σ, then Φ breaks off coordinating and starts producing inconsistent information.] Since $\langle \Psi, \mathbf{A} \rangle$ is self-centered, Ψ enumerates with Φ an environment for some $\mathcal{A} \in \mathbf{A}$ via some complete assignment h. On the other hand, Φ starts with σ and then copies Ψ's moves starting from the first of Ψ's moves. Then Φ enumerates with Ψ an environment for $\mathcal{A}$ via h, possibly after a finite number of errors equal to the number of elements in σ that are not true in $\mathcal{A}$ via h. Observe that $\mathcal{A} \in \mathbf{B}$, since $\mathbf{B} = \mathbf{A}$. It follows that $\langle \Psi, \mathbf{A} \rangle$ X-coordinates with $\langle \Phi, \mathbf{B} \rangle$.

Now define Φ' such that:

- (a) $\Phi'(\emptyset) = \sigma'_0$;
- (b) for all $\tau \in SEQ$ with $|\tau| = i > 0$, if $|\tau| < |\sigma'| - 1$ and $\tau = \overline{\Psi(\sigma'|_{i-1})}$, then $\Phi'(\tau) = \sigma'_{i+1}$;
- (c) for all other $\tau \in SEQ$, $\Phi'(\tau) = \Phi(\tau)$.

[In other words, Φ' starts with σ' if the first $|\sigma'| - 1$ moves by the opponent equal $\langle \Psi, \mathbf{A} \rangle$'s moves. Otherwise, Φ' moves as Φ.] Observe that Φ is a finite variant of Φ'. We claim that $\langle \Psi, \mathbf{A} \rangle$ does not X-coordinate with $\langle \Phi', \mathbf{B} \rangle$. To see this, observe that Φ' starts with σ', since $\langle \Phi', \mathbf{B} \rangle$'s opponent is $\langle \Psi, \mathbf{A} \rangle$ [so last condition (b)

above applies]. From the definition of Φ' it follows that for all $\tau \in SEQ$ with $|\tau| > |\sigma|$, $\Phi'(\tau) = \Phi(\tau)$. Thus as soon as $\Psi(\sigma) \neq \Psi(\sigma')$ Φ' outputs $\neg(v_0 \doteq v_0)$ forever after. Since $\Psi(\sigma) \neq \Psi(\sigma')$ by non-blindness, it follows that $\langle \Psi , \mathbf{A} \rangle$ and $\langle \Phi' , \mathbf{B} \rangle$ cannot X-coordinate. ■

The converse of Proposition (20) is false. Indeed:

(21) PROPOSITION: [AM01, Ago01b] There is a blind self-centered basic agent who is not X-forgiving.

Proof: Let basic agent $\langle \Psi_{\mathcal{A}} , \mathbf{A} \rangle$ with $\mathcal{A} \in \mathbf{A}$ be blind and such that $\Psi_{\mathcal{A}}$ enumerates with any basic agent an environment e for $\mathcal{A}$ via some complete assignment h and in such a way that every basic formula true in $\mathcal{A}$ via h occurs exactly once in e. Observe that $\langle \Psi_{\mathcal{A}} , \mathbf{A} \rangle$ is self-centered. Let e' denote an environment such that $range(e') = range(e)$. We define basic agents $\langle \Phi , \mathbf{B} \rangle$, $\langle \Phi' , \mathbf{B} \rangle$ as follows.

(a) $\mathbf{B} = \mathbf{A}$.

(b) For all $\sigma \in SEQ$, $\Phi(\sigma) = e'_{|\sigma|}$.

[Thus Φ enumerates e'.]

(c) For all $\sigma \in SEQ$, $\Phi'(\sigma) = {}_0|e'_{|\sigma|}$.

[Thus Φ' enumerates environment e' but the first element.]

Observe that Φ is a finite variant of Φ'. From (a) and (b) it follows that $\langle \Psi_{\mathcal{A}} , \mathbf{A} \rangle$ X-coordinates with $\langle \Phi , \mathbf{B} \rangle$. On the other hand, from (a) and (c) it follows that $\langle \Psi_{\mathcal{A}} , \mathbf{A} \rangle$ does not X-coordinate with $\langle \Phi , \mathbf{B} \rangle$, since ${}_0|e'$ is not an environment for $\mathcal{A}$. ■

7 Competence of Basic Agents

A general question to ask on any paradigm of coordination concerns the existence of classes Σ of agents such that the coordination competence of each agent in a class cannot be strictly improved. In this section, we exhibit a class of total, self-centered basic agents that has the required property for X-coordination. More precisely, we show that the X-coordination competence of any total, self-centered basic agent cannot be improved by any total basic agent with the same background knowledge.

We rely on the following sense of "coordination competence." (Recall that X is any of the prefixes in (16).[6])

(22) DEFINITION: The X-**coordination competence** of a basic agent $\langle \Psi , \mathbf{A} \rangle$ is the set

$$X\text{-}scope(\Psi_{\mathbf{A}}) = \{\langle \Phi , \mathbf{B} \rangle \in \Lambda^b \mid \Psi \rightleftharpoons_X \Phi\}.$$

[6]The results below apply to a wider class of paradigms of coordination. Some paradigms that are not described in this paper can be found in [Ago01b].

(23) DEFINITION: Basic agents $\langle \Psi, \mathbf{A} \rangle$, $\langle \Phi, \mathbf{A} \rangle$ are **distinct** just in case $\Psi(\sigma) \neq \Phi(\sigma)$ for some $\sigma \in SEQ$.

(24) PROPOSITION: For all distinct, self-centered basic agents $\langle \Psi, \mathbf{A} \rangle$, $\langle \Psi', \mathbf{A} \rangle$, there is a basic agent such that $\langle \Psi, \mathbf{A} \rangle$ X-coordinates with and that $\langle \Psi', \mathbf{A} \rangle$ does not.

Proof: Let self-centered basic agents $\langle \Psi, \mathbf{A} \rangle$ and $\langle \Psi', \mathbf{A} \rangle$ be distinct. Then there is $\sigma \in SEQ$ such that $\Psi(\sigma) \neq \Psi'(\sigma)$.

(25) Define ability $\Phi : SEQ \longrightarrow \mathcal{L}_{basic}$ such that:

(a) Φ starts with σ;

(b) for all $\tau \in SEQ$ with $|\tau| = |\sigma|$, $\Phi(\tau) = (v_0 \doteq v_0)$;

(c) for all $\tau \in SEQ$ with $|\tau| > |\sigma|$, if $\tau_{|\sigma|} = \Psi(\sigma)$ then $\Phi(\tau) = \Psi(\tau|_{|\tau|-|\sigma|})$;

(d) for all $\tau \in SEQ$ with $|\tau| > |\sigma|$, if $\tau_{|\sigma|} \neq \Psi(\sigma)$ then $\Phi(\tau) = \neg(v_0 \doteq v_0)$.

[In other words, Φ starts with σ; then, if there is no previous move by any opponent basic agent to look at, Φ moves "safe". Otherwise, if an opponent basic agent "agrees" with basic agent $\langle \Psi, \mathbf{A} \rangle$ on σ, then Φ starts copying the opponent's moves from the first one. If the opponent basic agent "disagrees" with $\langle \Psi, \mathbf{A} \rangle$ on σ, then Φ breaks off coordination and starts producing inconsistent information.] We claim that:

(a) $\langle \Psi, \mathbf{A} \rangle$ X-coordinates with $\langle \Phi, \mathbf{A} \rangle$.

(b) $\langle \Psi', \mathbf{A} \rangle$ does not X-coordinate with $\langle \Phi, \mathbf{A} \rangle$.

Proof of claim (a): Since $\langle \Psi, \mathbf{A} \rangle$ is self-centered, Ψ enumerates with Φ an environment for some $\mathcal{A} \in \mathbf{A}$ via some complete assignment h. On the other hand, Φ starts with σ and then copies Ψ's moves starting from the first of Ψ's moves. Then Φ enumerates with Ψ an environment for $\mathcal{A}$ via h, possibly after a finite number of errors equal to the number of elements in σ that are not true in $\mathcal{A}$ via h. It follows that $\langle \Psi, \mathbf{A} \rangle$ X-coordinates with $\langle \Phi, \mathbf{A} \rangle$. ▮

Proof of claim (b): Consider the interaction sequence of $\langle \Psi', \mathbf{A} \rangle$ and $\langle \Phi, \mathbf{A} \rangle$. Since Φ starts with σ, at stage $|\tau| = |\sigma| + 1$ of the interaction sequence $\tau_{|\sigma|} = \tau_{|\tau|-1} = \Psi'(\sigma)$. Since $\Psi'(\sigma) \neq \Psi(\sigma)$, from (25)($d$) it follows that for all $\tau \in SEQ$ with $|\tau| > |\sigma|$, $\Phi(\tau) = \neg(v_0 \doteq v_0)$. Then $\langle \Psi, \mathbf{A} \rangle$ does not X-coordinate with $\langle \Phi', \mathbf{A} \rangle$. ■

(26) COROLLARY: Let $\langle \Psi, \mathbf{A} \rangle$ be any total self-centered basic agent. Then there is no total basic agent $\langle \Psi', \mathbf{A} \rangle$ such that $X\text{-}scope(\Psi_{\mathbf{A}}) \subset X\text{-}scope(\Psi'_{\mathbf{A}})$.

Proof: Suppose for a contradiction that distinct, self-centered basic agents $\langle \Psi, \mathbf{A} \rangle$ and $\langle \Psi', \mathbf{A} \rangle$ are such that $X\text{-}scope(\Psi_{\mathbf{A}}) \subseteq X\text{-}scope(\Psi'_{\mathbf{A}})$. By the definition of X-coordination competence, it follows that for all basic agents $\langle \Phi, \mathbf{A} \rangle$, if $\langle \Psi, \mathbf{A} \rangle$ X-coordinates with $\langle \Phi, \mathbf{A} \rangle$ then $\langle \Psi', \mathbf{A} \rangle$ X-coordinates with $\langle \Phi, \mathbf{A} \rangle$. By Proposition (24), contradiction. ■

That is, the competence of a total, self-centered basic agent cannot be strictly improved.

8 Related Work and Conclusion

A preliminary discussion of absolute coordination is presented in [Ago00]. Local coordination paradigms od Section 5.1 involve common knowledge; see for instance [Gea94, Bar88, FHMV95] and [MS97]. In particular, it is not excluded though far from proved that $l\,s^{\circ}$-coordination can be used to model situations of collective choices so as to generalize [OSW87]. The criterion of sf°-coordination of Definition (15) is a special version of the criterion discussed in [AdJMed, AdJM01]. In particular, the two versions are equivalent if we assume $s = t$ in Definition (15)(a). Many ideas of Sections 6 and 7 are a re-formulation of ideas that appear in [MO99, Sec. 3.1, Sec. 5]. More precisely, compare Definition (19) of blind and forgiving basic agents with [MO99, Def. (22)]. Proposition (20) is an extension to X-coordination of Montagna and Osherson's Prop. (25); our Proposition (21) is their Prop. (23) and Proposition (24) extends Prop. (15).

In summary, in this paper we presented a formal framework suitable to investigate pairwise coordination over sets of agents. Both absolute and contextual coordination may be studied therein. As the results of this paper, we answered a number of sample problems.

Many other problems than those considered in this paper may be investigated in deep by modifying components like "agent," "dynamics," and "succes" according to the kind of coordination one is interested in. Problems of meaning negotiation and matching, composability, and semantic interoperability are some examples we are interested in related to the problem of coordination, that we naturally addressed in this paper. We leave them open for future work.

As the ideas and techniques become better developed, we expect that general principles will emerge from theory development that can be used in other research areas, for example distributed knowledge management and teamwork.

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